Method 2: Solving Systems by Substitution

Process:
1. Solve one of the equations for one of the variables.
2. Substitute the expression into the other equation and solve.
3. Substitute the result into the "solved equation" from step 1.
4. Check!

Example 1:

$$\begin{cases} 3x + y = -2 \ (1) \\ 4x + y = -4 \ (2) \end{cases}$$

Step 1: Solve ① for y.

① $y = -3x + (-2)$

Step 2: Substitute $-3x + (-2)$ for y.

② $4x + (-3x) + (-2) = -4$

$x + (-2) = -4$

$x = -2$

Step 3: Substitute $x = -2$ into ①.

① $y = -3(-2) + (-2)$

$y = 6 + (-2)$

$y = 4$

Step 4: check $(-2, 4)$

① $3(-2) + 4 = -2$

$-6 + 4 = -2$ ✓

② $4(-2) + 4 = -4$

$-8 + 4 = -4$ ✓

Solution: $(-2, 4)$

Example 2:

$$\begin{cases} -3x+2y=0 & ① \\ x+5y=17 & ② \end{cases}$$

Step 1: Solve ② for x.

② $x = -5y+17$

Step 2: Subst. $-5y+17$ for x in ①.

① $-3(-5y+17)+2y=0$

$15y+(-51)+2y=0$

$17y=51$

$y=3$

Step 3: Subst. 3 for y in ②.

② $x=-5(3)+17$

$x=-15+17$

$x=2$

Step 4: Check $(2,3)$

① $-3(2)+2(3)=0$

$-6+6=0$ ✓

② $2+5(3)=17$

$2+15=17$ ✓

Solution: $(2,3)$

Example 3:

$$\begin{cases} 3x+5y=15 & ① \\ y=-\frac{3}{5}x+3 & ② \end{cases}$$

Step 1: ② already solved for y.

Step 2: Subst. $-\frac{3}{5}x+3$ for y in ①.

① $3x+5\left(-\frac{3}{5}x+3\right)=15$

$3x+(-3x)+15=15$

$15=15$

Always True → Same Line

All solutions on $y=-\frac{3}{5}x+3$

Example 4:

$$\begin{cases} y = \frac{2}{3}x - 1 & (1) \\ 2x - 3y = -3 & (2) \end{cases}$$

Step 1: ① is already solved for y.

Step 2: Subst. $\frac{2}{3}x-1$ for y in ②.

② $2x-3\left(\frac{2}{3}x-1\right)=-3$

$2x+(-2x)+3=-3$

$3=-3$

Always False → Parallel Lines

No Solution

Example 5:

The perimeter of a rectangle whose length is two more than its width is 32 cm. Find the dimensions of the rectangle.

L = # of cm (length)

W = # of cm (width)

$$\begin{cases} L = W+2 & (1) \\ 2L+2W=32 & (2) \end{cases}$$

Step 1: Done

Step 2: ② $2(W+2)+2W=32$

$2W+4+2W=32$

$4W=28$

$W=7\text{cm}$

Step 3: ① $L=7+2$

$L=9\text{cm}$

Step 4: Chk ① $9=7+2$ ✓

② $2(9)+2(7)=32$

$18+14=32$ ✓

The dimensions of the rectangle are 7cm by 9cm.

Example 6:

An 18 question test is worth 50 points. It contains only 2-pt and 4-pt questions. How many of each question are on the test?

T = # of 2-pt questions
F = # of 4-pt questions

$$\begin{cases} T + F = 18 \ (1) \\ 2T + 4F = 50 \ (2) \end{cases}$$

Step 1: ① $T = -F + 18$

Step 2: ② $2(-F+18) + 4F = 50$

$-2F + 36 + 4F = 50$

$2F = 14$

$F = 7$ questions

Step 3: ① $T = -7 + 18$

$T = 11$ questions

Step 4: check ① $11 + 7 = 18$ ✓

② $2(11) + 4(7) = 50$

$22 + 28 = 50$ ✓

The test contains 11 2-pt questions and 7 4-pt questions.

Example 7:

An office supply store sells two types of fax machines. They charge $150 for one of the machines and $225 for the other. If the company sold 22 fax machines for a total of $3900 last month, how many of each type were sold?

C = # of cheaper machines
E = # of expensive machines

$$\begin{cases} C + E = 22 \ (1) \\ 150C + 225E = 3900 \ (2) \end{cases}$$

Step 1: ① $C = -E + 22$

Step 2: ② $150(-E+22) + 225E = 3900$

$-150E + 3300 + 225E = 3900$

$75E = 600$

$E = 8$ machines

Step 3: ① $C = -8 + 22$

$C = 14$ machines

Step 4: check ① $14 + 8 = 22$ ✓

② $150(14) + 225(8) = 3900$

$2100 + 1800 = 3900$ ✓

The store sold 14 of the $150 machines and 8 of the $225 machines.